

Electroproduction of electron-positron pair in a medium

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Abstract

The process of electron-positron pair creation by a high-energy electron in a medium is analyzed. The spectral distribution over energies of created particles is calculated for the direct and cascade mechanisms of the process. The Coulomb corrections are included. The new formulation of the equivalent photons method is developed which takes into account the influence of multiple scattering. It is shown the effects of multiple scattering can be quite effectively studied in the process under consideration.

1 Introduction

A high-energy electron passing through a medium produces the electron-positron pair side by side with the radiation. The electroproduction process possesses many important peculiarities which will be discussed below.

Generally speaking one has to consider two mechanisms of the process, The first one is the direct(one-step) electroproduction of pair via the virtual intermediate photon. The second one is the cascade(two-step) process when the electron emits the real photon at collision with one nucleus which converts into the pair on another nucleus. The interrelation of two mechanisms depends on the target thickness l since the probability of the direct process is proportional to l while the probability of the cascade process is proportional to $l^2/2$. These contributions have comparable values when the ratio l/L_{rad} (L_{rad} is the radiation length) is of the order of a few percent.

For the direct process the differential, over the energies of produced particles $\varepsilon_+, \varepsilon_-$, probability was found by Kelner [1] in the lowest over $Z\alpha$ (Z is the charge of a nucleus, $\alpha=1/137$) order of the perturbation theory (for the derivation see also Sec.26 in [2]). For the direct process there are two types of diagrams: one-photon diagram when the pair is created by one virtual photon emitted from the electron at collision with a nucleus, and two-photon diagrams when the pair is created by two virtual photons connected with the initial electron and a nucleus. In the present paper the Coulomb corrections are included in the probability contributions of both types of diagrams.

In the soft part of created particles spectrum ($\varepsilon_-, \varepsilon_+ \ll \varepsilon$, ε is the energy of the initial electron) one can neglect the one-photon contribution and the two-photon contribution can be obtained using the equivalent photons method within the logarithmic accuracy (see Appendix B in [3]). At low enough energies $\varepsilon_-, \varepsilon_+$ the multiple scattering of the initial electron results in distortion of the spectral distribution of the equivalent photons which can lead to a modification of the process probability. It is shown the effect of multiple scattering can be quite effectively studied in electroproduction process. Just as in the radiation process the effect can be observed in the soft part of spectrum.

In Sec.2 the probabilities of the direct process are presented. The new formulation of the equivalent photons method is given which includes the influence of multiple scattering. This method permits to find the alteration in the soft part of spectral distribution of created particles. In Sec.3 the probabilities of the cascade process taking into account the multiple scattering are considered. The probabilities, differential over one of created particle energy, are analyzed in Sec.4

2 Direct electroproduction probability with Coulomb corrections

The Coulomb corrections to the direct electroproduction probability can be found using the method outlined in the review [4] (see also Appendix A in [5]). The probability for the one-photon diagrams contribution with the Coulomb corrections taken into account in the case of complete screening has the form (the system $\hbar = c = 1$ is used)

$$\begin{aligned} \frac{dw_1}{dzdy} = \frac{\alpha l}{\pi L} \frac{1-y}{y^2} & \left\{ \left(A_1 \ln(1+\xi) + B_1 + C_1 \frac{\xi}{1+\xi} \right) \left(1 + \frac{\ln(1+\frac{1}{\xi})}{2L_0} \right) \right. \\ & + \frac{1}{2L_0} \left[A_1 \text{Li}_2 \left(\frac{\xi}{1+\xi} \right) + B_1 \frac{\ln(1+\xi)}{\xi} + C_1 \frac{2}{3} \frac{\xi}{1+\xi} \right. \\ & \left. \left. + \frac{2}{9} \left(\left(1 - \beta \left(1 + \frac{1}{\xi} \right) \right) \ln(1+\xi) + \beta \right) \right] \right\}, \end{aligned} \quad (1)$$

where m is the electron mass, $\text{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t} dt$ is the Euler dilogarithm,

$$\begin{aligned} z = \frac{\varepsilon_+}{\varepsilon}, \quad y = \frac{\omega}{\varepsilon}, \quad \omega = \varepsilon_+ + \varepsilon_-, \quad \frac{1}{L} = \frac{4Z^2\alpha^3 n_a L_0}{m^2}, \quad L = L_{\text{rad}} \left(1 + \frac{1}{18L_0} \right), \\ L_0 = \ln(183Z^{-1/3}) - f(Z\alpha), \quad f(x) = \sum_{n=1}^{\infty} \frac{x^2}{n(n^2 + x^2)}, \\ A_1 = \xi \left(\frac{1}{2\beta} - 1 \right) - \frac{4}{3}\beta \left(1 + \frac{1}{\xi} \right) - \frac{2}{3}, \quad B_1 = \frac{4}{3}\beta + \xi, \\ C_1 = \frac{4}{3}\beta + \frac{1}{3} \left(1 + \xi \left(1 - \frac{1}{2\beta} \right) \right), \quad \beta = \frac{z(y-z)}{y^2}, \quad \xi = \frac{z(y-z)}{1-y}, \end{aligned} \quad (2)$$

where n_a is the number density of atoms in the medium, $f(Z\alpha)$ is the Coulomb correction. It should be noted that in Eq.(1) at $\xi \ll 1$ a mutual compensation occurs in the braces and the expression in the braces becomes proportional to ξ .

The probability for the two-photon diagrams contribution with the Coulomb corrections taken into account in the case of complete screening has the form

$$\begin{aligned} \frac{dw_2}{dzdy} = \frac{\alpha l}{\pi L} \frac{1-y}{y^2} & \left\{ \left(A_2 \ln(1+\frac{1}{\xi}) + B_2 + C_2 \frac{1}{1+\xi} \right) \left(1 + \frac{\ln(1+\xi)}{2L_0} \right) \right. \\ & + \frac{1}{2L_0} \left[A_2 \text{Li}_2 \left(\frac{1}{1+\xi} \right) + B_2 \xi \ln(1+\frac{1}{\xi}) + \frac{2}{3} \frac{C_2}{1+\xi} \right. \\ & \left. \left. - \frac{2}{9} \left((\beta + \xi(1+\beta)) \ln \left(1 + \frac{1}{\xi} \right) - \beta \right) \right] \right\}, \end{aligned} \quad (3)$$

where

$$A_2 = 1 - \frac{4}{3}\beta + \xi \left(\frac{1}{2\beta} + \frac{2}{3}(1-2\beta) \right), \quad B_2 = \frac{4}{3}\beta - 1,$$

$$C_2 = \frac{1}{3} \left(4\beta - 1 - \xi \left(1 + \frac{1}{2\beta} \right) \right). \quad (4)$$

The probabilities Eqs.(1), (3) are calculated within the power accuracy (neglected terms $\propto m/\varepsilon_+, m/\varepsilon_-$).

In the soft part ($y \sim z \ll 1$) of spectral distribution w_2 one can include the influence of multiple scattering on the initial electron, the result is

$$\frac{dw_2^m}{dzdy} = \frac{\alpha l}{\pi L} \frac{1-y}{y^2} \left[\Phi_2 + \frac{\pi^2}{12L_0} \left(1 - \frac{4}{3}\beta \right) + \frac{1}{3}(4\beta - 1) \left(1 + \frac{1}{3L_0} \right) \right], \quad (5)$$

where

$$\begin{aligned} \Phi_2 &= P(y, z) \left(\ln \frac{1}{\xi} - 1 - \ln(1 + \nu_1) \right), \quad P(y, z) = 1 - \frac{4}{3}\beta \left(1 + \frac{1}{12L_0} \right), \\ \nu_1 &= \frac{\varepsilon(1-y)}{\varepsilon_e y}, \quad \frac{m^2}{\varepsilon_e} = \frac{16\pi Z^2 \alpha^2 n_a L_0}{m^2} = \frac{4\pi}{\alpha L}, \end{aligned} \quad (6)$$

Here the function Φ_2 describes the pair creation probability in the equivalent photons method. Appearance of the term $\ln(1 + \nu_1)$ in the function Φ_2 is connected with expansion of the characteristic equivalent photon emission angles ϑ_c due to the multiple scattering of the initial electron. Let us consider this item in detail. The density of equivalent photon can be presented as (see Eq.(B.7) in [3])

$$n(y, z) = \frac{\alpha}{\pi} \frac{1-y}{y} \left[\left(1 + \frac{y^2}{2(1-y)} \right) \ln \frac{Q^2(y, z)}{q_c^2} - 1 \right], \quad (7)$$

where $Q^2 = m^2 \omega^2 / (\varepsilon_+ \varepsilon_-)$ is the squared minimal momentum transfer which is necessary for the photon with the energy ω to create the pair with the energies $\varepsilon_+, \varepsilon_-$. In absence of multiple scattering q_c^2 is defined by the kinematics of the virtual photon emission from the initial electron

$$q_c^2 = q_{min}^2 = \frac{m^2}{\varepsilon^2} \frac{\omega^2}{1-y}, \quad \frac{Q^2}{q_{min}^2} = \frac{\varepsilon^2(1-y)}{\varepsilon_+ \varepsilon_-} = \frac{1-y}{z(y-z)} = \frac{1}{\xi}. \quad (8)$$

Taking into account the multiple scattering one has $\varepsilon^2 \vartheta_c^2 / m^2 = 1 + \nu_1$ (see e.g. Eqs.(2.10), (2.25), (2.26) in [4]). So the equivalent photon spectrum at $y \sim z \ll 1$ can be written as

$$q_c^2 = \frac{\omega^2 \vartheta_c^2}{1-y}, \quad n(y, z) dy = \frac{\alpha}{\pi} \frac{1-y}{y} \left[\ln \frac{1}{\xi} - 1 - \ln(1 + \nu_1) \right] dy. \quad (9)$$

The differential probability of the pair creation by a photon in the case of complete screening has the form

$$\frac{dw_p}{dz} = \frac{l}{L} \frac{P(y, z)}{y}, \quad (10)$$

and we find that the term $\propto \Phi_2$ in Eq.(5) corresponds to the equivalent photon method. The contribution of the multiple scattering is included in Eq.(6) within the logarithmic accuracy. Let us note that for heavy elements the value ε_e is of the order of a few TeV (e.g. $\varepsilon_e=2.73$ TeV for tungsten, $\varepsilon_e=2.27$ TeV for iridium), so for the electron energy of a few hundreds GeV, $\nu_1 \sim 1$ at $y \sim 1/10$.

3 Cascade electroproduction probability

It is known that the multiple scattering distorted the radiation spectrum when $\nu_1 \geq 1$ or the photon energy $\omega \leq \omega_c = \varepsilon^2/(\varepsilon + \varepsilon_e)$ (this is the Landau-Pomeranchuk-Migdal(LPM) effect), while for the pair creation process the LPM effect distorted the spectrum of created pair only when $\omega \geq \omega_e = 4\varepsilon_e$ (see [4]). So for available electron energies one have to take into account the multiple scattering in the cascade electroproduction process for the radiation part only. Than the spectral distribution of cascade process taking into account the multiple scattering of the initial electron has the form

$$\begin{aligned} \frac{dw_c}{dzdy} = & \left(\frac{\alpha m^2 l}{4\pi \varepsilon_e} \right)^2 \frac{\varepsilon_e P(y, z)}{\varepsilon y(1-y)} \text{Im} \left\{ y^2 \left[\ln p - \psi \left(p + \frac{1}{2} \right) \right] \right. \\ & \left. + \left[2(1-y) + y^2 \right] \left[\psi(p) - \ln p + \frac{1}{2p} \right] \right\}, \end{aligned} \quad (11)$$

where $p = \sqrt{i}/(2\nu_1)$, $\psi(x)$ is the logarithmic derivative of the gamma function (see Eq.(2.17) in [4]), the value ν_1 is defined in Eq.(6). In the term describing radiation ($\text{Im} \{ \}$) the term $\propto 1/L_0$ is neglected. The contribution of this term doesn't exceed a few per cent under considered conditions. This term is given by Eq.(2.33) in [5]) and can be included in the relevant case. In the case when the multiple scattering of the initial electron may be neglected ($\omega \gg \omega_c$, $\nu_1 \ll 1$, $|p| \gg 1$) the spectral distribution of cascade process is

$$\frac{dw_c^{QED}}{dzdy} = \frac{l^2}{L^2} \frac{P(y, z)}{2y^2} \left[y^2 + \frac{4}{3}(1-y) \left(1 + \frac{1}{12L_0} \right) \right]. \quad (12)$$

Here the terms $\propto 1/L_0$, neglected in Eq.(11), are taken into account.

In the cascade process side by side with radiation inside of a target one has to take into account the boundary radiation. Using Eq.(3.12) in [4] (with regard for the factor 1/2 since photons emitted at fly out of a target can't create the pair) and assuming that $(\omega_0/(my))^2 \ll 1 + \nu_1$, where ω_0 is the plasma frequency (in any medium $\omega_0/m \leq 10^{-4}$) one has

$$\frac{dw_b}{dzdy} = \frac{\alpha^2 m^2 l}{4\pi^2 \varepsilon_e} \frac{1-y}{y^2} P(y, z) \ln(1 + \nu_1) \quad (13)$$

Putting together this probability and the probability Eq.(5) we have that the terms with $\ln(1 + \nu_1)$ are canceled. So the sum of contributions of the equivalent and boundary photons doesn't depend on multiple scattering.

At photon energy ω decreasing starting with $\omega = \omega_c$ ($\nu_1 = 1$) the influence of multiple scattering on the radiation process becomes significant. At this energy the estimation of the interrelation of the different contributions is

$$\frac{dw_2^m + dw_b}{dw_c} = \frac{dw_2}{dw_c} \sim \frac{2L_{rad}}{l} \frac{\alpha}{\pi} \ln\left(\frac{\varepsilon_e}{\varepsilon}\right) \quad (14)$$

With further decreasing of photon energy the relative contribution of the cascade process is dropping both because of the logarithmic growth of the probability dw_2 and because of the suppression of the real photon emission probability due to the multiple scattering of projectile (the LPM effect). In this interval of photon energies (but for $\omega \gg m$) in the case when the value l is low enough the thickness of the target can become less than the photon (virtual or real) formation length

$$l_f = \frac{2}{\omega \vartheta_c^2} = \frac{2\varepsilon^2}{m^2 \omega (1 + \nu_1)}, \quad \frac{l}{l_f} \simeq \frac{l}{L_{rad}} \frac{2\pi(1 + \nu_1)}{\alpha \nu_1^2}. \quad (15)$$

In this limiting case the contribution of real photons into the process probability vanishes and only the contribution of virtual photons remains. These photons build up outside a target where there is no influence of the multiple scattering.

It should be noted that with photon energy decreasing the formation length of pair creation by a photon inside target is also decreasing ($l_p = 2\omega/m^2$) and we can be out of the complete screening limit. However in this energy interval ($y \leq 10^{-3}$) the equivalent photons method is applicable within the quite satisfactory accuracy and the cross section of the photo-process is known for arbitrary screening (see [6, 2]).

4 Partially integrated electroproduction probability

The probability of electroproduction differential over one of created particle energy only is of evident interest. It can be obtained by integration of the found probabilities over y ($z \leq y \leq 1$). For $z \ll 1$ the main contribution into the integral gives the region $y \sim z \ll 1$ (with the exception of the contribution of one-photon diagrams which can be neglected in this energy region). For the ratio $r = dw_1/dw_2$ one has $r = 0.011$ at $z = 0.1$, $r = 0.042$ at $z = 0.2$ and $r = 0.24$ at $z = 0.5$. Using Eq.(6) at $\nu_1 = 0$ and conserving the main term of decomposition over z one obtains for the summary contribution of the equivalent and boundary photons

$$\begin{aligned} \frac{dw_b}{dz} + \frac{dw_2^m}{dz} &= \frac{dw_2}{dz} = \frac{2\alpha}{\pi z} w_p \left(\ln \frac{1}{z} - \frac{1}{2} + \delta \right), \\ w_p &= \frac{28}{9} \frac{Z^2 \alpha^3}{m^2} \ln_a \left(L_0 - \frac{1}{42} \right) \simeq \frac{7l}{9L_{rad}}, \quad \delta = \frac{\pi^2}{24L_0} - \frac{1}{14} \left(1 + \frac{1}{3L_0} \right). \end{aligned} \quad (16)$$

Here w_p is the probability of pair creation by a photon in the target with thickness l in the case of complete screening presented within power (relativistic) accuracy, neglected terms are $\sim m/\varepsilon_+$. The quantity δ is the correction to the equivalent photons method. This correction is small numerically: e.g. for heavy elements ($L_0 \simeq 3.5$, $\delta \simeq 1/25$), for Ge $L_0 \simeq 4$, $\delta \simeq 1/40$.

For the cascade process contribution in the region $z \ll 1$, $\nu_1(z) \ll 1$ one gets

$$\frac{dw_c}{dz} = \frac{2l}{3Lz} \left(1 + \frac{1}{12L_0}\right) w_p \simeq \frac{14}{27z} \left(\frac{l}{L_{rad}}\right)^2. \quad (17)$$

The spectral distributions of created positrons reflect the spectral distribution of photons (up to common factor w_p in Eq.(16) and $w_p/2$ in Eq.(17)).

When the parameter ν_1 is large the asymptotic regime for the radiation probability (see Eqs.(2.31),(2.32) of [4]) is realized at very high value of ν_1 . Because of this one has to use Eq.(11) directly. Conserving the main terms over $1/y$ we find

$$\frac{dw_c}{dz} \simeq \left(\frac{l}{L_{rad}}\right)^2 \frac{2\varepsilon_e}{\varepsilon} \int_z^1 \left(1 - \frac{4z(y-z)}{3y^2}\right) \text{Im} \left(\psi(x) - \ln x + \frac{1}{2x}\right) \frac{dy}{y}, \quad (18)$$

where $x = x(y) = \sqrt{i\varepsilon_e y/(4\varepsilon)}$.

Since the spectral distribution has the general factor $1/z$ its characteristic properties at variation of z over a few order of magnitude one can track analyzing the function zdw/dz . For the same reason in Figs 2 and 3 in the region under consideration ($z \leq 0.2$) the dependence of this combination on the positron energy $\varepsilon_+(z = \varepsilon_+/\varepsilon)$ is shown. The probabilities Eqs.(3), (11) were used in calculation.

In the Fig.1 the spectral density dw/dz for thin targets is shown. The difference between curves 1 and 2 is due to the influence of multiple scattering. Since the targets are quite thin, the difference is still small especially for $l = 170 \mu m$. The integral $n_{12} = \int_{z_1}^{z_2} (dw/dz) dz$ gives the number of positron per one initial electron in the energy interval $\varepsilon_{z_1} - \varepsilon_{z_2}$. For thickness $l = 400 \mu m$ one has $n_{12} \simeq 9 \cdot 10^{-4}$ for the positron energies interval 0.5-5 GeV. The targets of mentioned thicknesses were used in the experiment NA63 carried out recently at SPS at CERN (for proposal see [7]).

The Fig.2 is another look on the process which permits to trace details of the pair creation mechanism. The curves 2,5 in the right part increase first tending to the asymptotic Eq.(17) and than decrease because of transition to the regime of Eq.(18) at the characteristic energy $z_c = y_c = \omega_c/\varepsilon = \varepsilon/\varepsilon_e = 0.01$. The curves 1,4 are described nearly completely by Eq.(16). The increase of combination zdw/dz is due to $\ln 1/z$. This contribution dominates in the summary combination in the left part of the spectrum for $l = 400 \mu m$ and in the whole spectrum for $l = 170 \mu m$ (curves 3,6). Because of this the relative influence of multiple scattering on the electroproduction process is falling.

The Fig.3 shows a different situation when the target is relatively thick. Evidently here the influence of multiple scattering spreads to the more wide positron

energy interval (the region where the curve 2 decreases). The cascade mechanism dominates for the positron energy higher than 10 GeV.

In the Fig.4 the difference between the curves 1 and 2 shows the influence of multiple scattering. This difference can be characterized by ratio (see Eqs.(3), (11), (12))

$$\Delta = \frac{dw_c^{QED} - dw_c}{dw_c + dw_2}. \quad (19)$$

In tungsten for the thickness $l = 0.03$ cm one has at the initial electron energy $\varepsilon = 50$ GeV for the created positron energy $\varepsilon_+ = 50$ MeV ($z = 0.001$) the value $\Delta \simeq 42\%$, and at the initial electron energy $\varepsilon = 300$ GeV for the created positron energy $\varepsilon_+ = 100$ MeV the value $\Delta \simeq 100\%$.

5 Conclusion

The process of electron-positron pair production by a high-energy electron traversing amorphous medium is investigated. It is shown that the soft part of created particle spectrum may reduced due to the multiple scattering of the initial electron. In the direct process (via the virtual intermediate photon) the equivalent photon spectrum is changed under the influence of multiple scattering. In the cascade process (via real intermediate photon) the multiple scattering distorted the photon spectrum inside a target. Besides, the contribution of the boundary photons appears. It is shown, within the logarithmic accuracy, that the change of the equivalent photon spectrum is canceled by the contribution of the boundary photons. As a result one has that the influence of multiple scattering may be neglected in the very thin targets ($l \leq 1\%L_{rad}$), where the direct process dominates in the soft part of photon spectrum. The different situation arises in a more thick target of heavy elements ($l \sim$ a few % of L_{rad}). For the initial electron energy in the range of hundreds GeV the multiple scattering substantially diminish the spectrum of created positrons in the range from hundreds MeV to a few GeV. This phenomenon can be used for further study of the influence of multiple scattering on higher order QED processes.

Acknowledgments

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Figure captions

Fig.1

The summary spectral distribution $dw/dz = dw_2/dz + dw_c/dz$ of pair electroproduction in amorphous germanium at the initial electron energy $\varepsilon = 180$ GeV. In the curves 1 and 3 (for two target thicknesses $l = 400 \mu m$ and $l = 170 \mu m$ respectively) the multiple scattering is taken into account (Eq.(11)), while in the curves 2 and 4 the multiple scattering is neglected (Eq.(12)).

Fig.2

The combination zdw/dz for the pair electroproduction probability in amorphous Ge at the initial electron energy $\varepsilon = 180$ GeV. The dotted curves 1 and 4 are the contributions of two-photon diagrams Eq.(3), the dashed curves 2 and 5 are the contributions of cascade process Eq.(11), the solid curves 3 and 6 are the sum of two previous contributions for two thicknesses $l = 400 \mu m$ and $l = 170 \mu m$ respectively. For convenience the ordinate is multiplied by 10^3 .

Fig.3

The combination zdw/dz for the pair electroproduction probability in amorphous tungsten at the initial electron energy $\varepsilon = 300$ GeV. The dotted curve 1 is the contribution of two-photon diagrams Eq.(3), the dashed curve 2 is the contribution of cascade process Eq.(3), the solid curve 3 is the sum of two previous contributions for the target thicknesses $l = 300 \mu m$ ($8.6\% L_{rad}$) For convenience the ordinate is multiplied by 10^3 .

Fig.4

The summary spectral distribution $dw/dz = dw_2/dz + dw_c/dz$ of the pair electroproduction in amorphous tungsten of the thickness $l = 300 \mu m$ ($8.6\% L_{rad}$) at the initial electron energy $\varepsilon = 50$ GeV. In the curve 1 the multiple scattering is taken into account (Eq.(11)), while in curve 2 the multiple scattering is neglected (Eq.(12)).







